

An improved quantitative measure of the tendency for volcanic ash plumes to form in water: implications for the deposition of marine ash beds

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Abstract

Laboratory experiments and numerical simulations have shown that volcanic ash particles immersed in water can either settle slowly and individually, or rapidly and collectively as particle-laden plumes. The ratio of timescales for individual and collective settling, in the form of analytical expressions, provides a dimensionless quantitative measure of the tendency for such plumes to grow and persist which has important implications for determining particle residence times and deposition rates. However, existing measures in the literature assume that collective settling obeys Stokes' law and is therefore controlled by the balance between gravitational forces and viscous drag, de-

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spite plume development actually being controlled by the balance between gravitational forces and inertial drag even in the absence of turbulence during early times. This paper presents a new measure for plume onset which takes into account the inertial drag-controlled (rather than viscous drag-controlled) nature of plume growth and descent. A parameter study comprising a set of numerical simulations of small-scale volcanic ash particle settling experiments highlights the effectiveness of the new measure and, by comparison with an existing measure in the literature, also demonstrates that the timescale of collective settling is grossly under-estimated when assuming that plume development is slowed by viscous drag. Furthermore, the formulation of the new measure means that the tendency for plumes to form can be estimated from the thickness and concentration of the final deposit; the magnitude and duration of particle flux across the water's surface do not need to be known. The measure therefore permits the residence times of particles in a large body of water to be more accurately and practically determined, and allows the improved interpretation of layers of volcanoclastic material deposited at the seabed.

Keywords: Ash plumes, Settling rates, Volcanoclastic deposits, Computational modelling, Numerical simulations, Vertical density currents

1. Introduction

Explosive volcanism generates vast quantities of small ash particles which can be transported over great distances, eventually depositing both on land and on the seabed to form particle layers (Carey and Schneider, 2011). These layers are a text-book example of isochroneity and have been used for strati-

6 graphic correlation of past eruption events (e.g. Ver Straeten (2004, 2008)),
7 allowing a wealth of information regarding their duration and frequency to
8 be determined. Furthermore, ash deposits can potentially preserve informa-
9 tion about the environmental conditions at the time of an event (Manville
10 and Wilson, 2004). However, the process behind the settling of ash and the
11 resulting formation of the particle layers is far from simple.

12 It was once assumed that the settling of ash in the deep sea occurred
13 passively such that particles always descend slowly and individually under
14 Stokes' law (Ledbetter and Sparks, 1979; Carey and Schneider, 2011), but
15 several field-based observations have provided contradictory evidence. For
16 example, following the 1991 eruption of Mount Pinatubo, ash fallout in the
17 South China Sea settled at speeds of over 2 cms^{-1} which is two to three orders
18 of magnitude greater than the calculated Stokes' law velocities of individual
19 particles (Wiesner et al., 1995). Through analogous laboratory experiments,
20 Carey (1997) set out to explore this apparent contradiction in timescales and
21 revealed the important role of vertical density currents in the rapid, collective
22 transportation of material to the seabed.

23 The generation of vertical density currents is a complex multiphase pro-
24 cess. Particles entering a body of water, either as fallout from ash clouds in
25 the atmosphere or from a pyroclastic density current, undergo abrupt decel-
26 eration as they cross the air-water interface. Initially, slow and individual
27 settling under Stokes' law ensues, allowing the particle concentration near
28 the surface to rapidly increase and form a layer of particle-rich water over
29 time. However, if the particle concentration in the layer is large enough
30 for the particles to affect each other's settling through drag reduction and

31 drifting such that the layer becomes gravitationally unstable, then finger-like
32 Rayleigh-Taylor instabilities eventually form along the interface between the
33 layer and the particle-free water below it. These instabilities grow exponen-
34 tially to form plumes — clouds of particles that settle rapidly and collectively
35 as vertical density currents.

36 Knowing whether plumes are likely to form, if at all, is important if one
37 wishes to better determine the timescale of settling from the surface to the
38 seabed. This can reveal information about the residence time of particles
39 in the water and therefore the extent to which ambient ocean currents re-
40 distribute volcanoclastic material as it settles (Carey and Schneider, 2011).
41 Similarly, knowing the rate of deposition can help determine the degree of
42 bioturbation of the growing particle layer by marine organisms (Bramlette
43 and Bradley, 1941). Plume formation also has implications for fossil preser-
44 vation and stratigraphy. Rapid sedimentation has long been recognised as a
45 means of increasing the likelihood that an organism could be preserved as a
46 fossil (Seilacher et al., 1985) and so ash plume formation can impact upon the
47 completeness of the fossil record. Perhaps one of the most celebrated and ge-
48 ologically significant examples of exceptional preservation beneath a marine
49 ash deposit is that of the Neoproterozoic Ediacaran biota in Newfoundland
50 which preserves some of the earliest metazoan fossils on Earth (Narbonne,
51 2005).

52 *1.1. Theoretical Considerations*

53 Quantitatively describing the tendency for plumes of particles to form in
54 an ambient fluid has been achieved in previous works (Marsh, 1988; Goldin,
55 2008; Carazzo and Jellinek, 2012) through a dimensionless number B . This is

56 defined in such a way that values of B less than or equal to unity imply that
 57 plumes do not form, whereas a value greater than unity implies favourable
 58 conditions for plume growth and persistence. In particular, existing dimen-
 59 sionless numbers have been defined by the ratio of timescales for individual
 60 particle settling under Stokes' law and collective settling as a gravitationally
 61 unstable plume, such that

$$B = \frac{\tau_{\text{individual}}}{\tau_{\text{collective}}}. \quad (1)$$

62 That is, given information about the current state of Rayleigh-Taylor insta-
 63 bilities, the time required for particles to reach that state through individual
 64 and collective settling modes can be approximated using analytical expres-
 65 sions. Clearly a value of $B \gg 1$ implies favourable conditions for plume
 66 formation and persistence since collective settling happens over a shorter
 67 timescale (e.g. days or weeks in the ocean) than individual settling (e.g.
 68 months), whereas a value of $B \approx 1$ implies that plumes cannot form since
 69 the timescales of individual and collective settling are of the same order of
 70 magnitude. Note that a value of $B < 1$ also implies that plumes cannot form,
 71 but when B is defined by the ratio of timescales this value has no physical
 72 meaning except for the case of hindered settling (Kuenen, 1968) which is
 73 not considered here. The parameters needed to compute these expressions
 74 include the particle concentration and the thickness of the particle-rich layer
 75 which often have to be estimated in practice. Alternatively, the measure
 76 can be re-formulated in terms of a critical layer thickness that must be at-
 77 tained in order for pluming to take place (discussed later). This only requires
 78 knowledge of the mass influx across the water's surface and particle diameter

79 which is often readily available during or after an eruption event.

80 One such formulation of B is the one derived by Marsh (1988) for the
 81 study of crystal settling in magma, denoted B_{vv} in this paper. This formu-
 82 lation is based on the assumption that both individual particles and plumes
 83 obey Stokes' law and are therefore controlled by the balance between gravita-
 84 tional forces (weight and buoyancy) and the viscous drag force (i.e. the drag
 85 arising from the friction between the descending particles/plumes and the
 86 ambient fluid), hence the use of the subscript vv to denote 'viscous-viscous'.
 87 The time taken for an individual (spherical) particle to settle through a layer
 88 of thickness h is therefore given by

$$\tau_{\text{individual}} = \frac{18h\mu_f}{(\rho_p - \rho_f)gd_p^2}, \quad (2)$$

89 where d_p is the particle diameter, g is the acceleration due to gravity, μ_f is
 90 the viscosity of the fluid phase, and ρ_f and ρ_p are the density of the fluid and
 91 particle phase, respectively (Stokes, 1851). The assumption that all particles
 92 have a perfect spherical shape is implicitly built-in to the timescale above
 93 through the Stokes drag coefficient. Furthermore, it has been shown (see for
 94 example Whitehead and Luther (1975); Goldin (2008)) that the timescale of
 95 collective settling is given by:

$$\tau_{\text{collective}} = \frac{18\mu_f}{\alpha_p(\rho_p - \rho_f)gh}, \quad (3)$$

96 where α_p is the volume fraction of particles in the layer. Taking the ratio of
 97 these two timescales yields the dimensionless number B_{vv} :

$$B_{vv} = \frac{\alpha_p h^2}{d_p^2}. \quad (4)$$

98 Further work by Carazzo and Jellinek (2012) derived similar non-dimensional
99 numbers for the scenario of volcanic ash settling through the atmosphere.
100 Coarse-grained ash and lapilli can settle individually with a particle Reynolds
101 number several orders of magnitude greater than that of fine ash (Bonadonna
102 et al., 1998), so three forms of B were derived using different expressions for
103 $\tau_{\text{individual}}$ to cover a wide range of individual particle settling regimes. How-
104 ever, none of these measures address the fact that plume growth and descent
105 are controlled by the balance between gravitational forces and the inertial
106 drag force (Dalziel et al., 2008; Bergantz and Ni, 1999). This inertial drag
107 force arises from the need for the plumes to accelerate and displace the sur-
108 rounding fluid, even in the absence of fluid viscosity, and dominates the
109 viscous drag force as shown by plume Reynolds numbers¹ much greater than
110 unity (Jacobs et al., 2013). At this point Stokes' law no longer holds even
111 if no turbulent effects are observed until the plumes are fully developed and
112 begin to mix, which has a significant impact on entrainment and settling
113 rates (Manville and Wilson, 2004). A measure which assumes that collective
114 particle settling is slowed by inertial drag (rather than viscous drag) may
115 therefore be more appropriate.

116 This paper presents a new measure of the tendency for particles to form
117 plumes and settle collectively which accounts for the fact that collective par-
118 ticle settling is slowed by inertial drag. The new non-dimensional number,
119 denoted B_{vi} , is derived by applying Stokes' law and a well-founded expression
120 for the growth rate of Rayleigh-Taylor instabilities (Youngs, 1984). The va-

¹The Reynolds number is a dimensionless quantity defined as the ratio of inertial to viscous drag force.

121 lidity of the measure for predicting the formation of plumes as particles settle
122 in water is then evaluated and compared against B_{vv} . This is accomplished
123 by (a) using data from the experiments by Carey (1997) which consider ash
124 particles settling through a water tank, and (b) performing a parameter study
125 through analogous numerical simulations with the multiphase computational
126 fluid dynamics (CFD) code Fluidity (Piggott et al., 2008; Davies et al., 2011;
127 Jacobs et al., 2013). The paper finishes with a discussion of the implications
128 and applications of the new measure, other geophysical scenarios where the
129 new measure could also be valid, and some concluding remarks. A list of
130 notation used throughout the paper is provided in Appendix A.

131 2. Derivation of the New Measure

132 To derive a measure of the tendency of plumes to form which takes into
133 account the fact that collective settling is slowed by inertial (rather than
134 viscous) drag, consider the growth of wave-like instabilities with maximum
135 amplitude δ at the interface between a particle-water layer of thickness h
136 and the particle-free water beneath it, as illustrated in Figure 1. The water
137 is treated as an incompressible fluid, and the particles have an idealised
138 spherical shape.

139 From Stokes' law, the timescale required for an individual spherical par-
140 ticle to settle through the layer of thickness h is given by (2) previously. A
141 timescale for the settling of a cloud of particles with a growing amplitude
142 δ can be derived from an ordinary differential equation describing the late-
143 time growth rate of Rayleigh-Taylor instabilities (Ristorcelli and Clark, 2004;
144 Youngs, 1984),

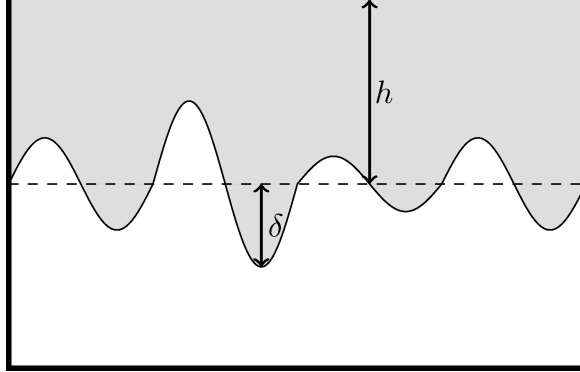


Figure 1: Illustration of particle plumes forming in a tank of water. The height of the particle-rich layer is denoted by h . The amplitude of the longest growing instability is denoted by δ .

$$\frac{d\delta}{dt} = 2\sqrt{\beta Atg\delta}, \quad (5)$$

145 where β is a dimensionless constant growth parameter, $At = \frac{\rho - \rho_f}{\rho + \rho_f}$ is the
 146 Atwood number, and ρ is the bulk density of the plume defined as $\rho =$
 147 $\alpha_f \rho_f + \alpha_p \rho_p$ where α_f is the volume fraction of the fluid. In this work,
 148 $\beta = 0.03$ which is within the range of values estimated by experimental
 149 and numerical techniques (Dimonte and Schneider, 2000; Dimonte et al.,
 150 2004). This expression can be readily integrated to provide an expression for
 151 $\tau_{\text{collective}}$, given by (Youngs, 1984)

$$\tau_{\text{collective}} = \sqrt{\frac{\delta}{\beta Atg}}. \quad (6)$$

152 Note that the initial condition $\delta(t = 0) = 0$ has been applied here. Although
 153 $t = 0$ is supposed to be the point at which the flow reaches self-similarity

154 (that is, when the flow behaviour appears the same on any scale) such that
 155 the initial condition becomes $\delta(t = 0) = \delta_0$ for some $\delta_0 > 0$, this work chooses
 156 $t = 0$ to correspond to the very start of the numerical simulation. This choice
 157 was shown *a posteriori* to still provide a consistently close approximation to
 158 the growth rate of the plumes across all simulations, even during very early
 159 times. Furthermore, this choice was made in order to be consistent with the
 160 expression for $\tau_{\text{individual}}$ and to avoid any ambiguity in deciding exactly when
 161 the flow becomes self-similar.

162 Taking the ratio of (2) and (6) yields the new dimensionless number

$$B_{vi} = \frac{18h\mu_f}{d_p^2} \sqrt{\frac{\alpha_p\beta}{(\rho + \rho_f)(\rho_p - \rho_f)\delta g}}. \quad (7)$$

163 It should be emphasised that this dimensionless quantity assumes that the
 164 ambient fluid is incompressible, and that individual particle settling is con-
 165 trolled by the balance between gravitational forces and viscous drag, whereas
 166 plume growth and descent (i.e. collective particle settling) is controlled by
 167 the balance between gravitational forces and inertial drag. Additional mea-
 168 sures can be derived for a compressible ambient fluid (Goldin, 2008), which
 169 is important for scenarios in which particles with a high initial momentum
 170 move through the atmosphere, and for different regimes of individual and col-
 171 lective particle settling. For completeness, the B_{ii} measure appropriate for
 172 very coarse-grained particles that settle individually at Reynolds numbers
 173 much greater than unity, implying that the inertial drag force dominates vis-
 174 cous drag, is presented in Appendix B. However, this measure is not tested
 175 here.

176 3. Numerical Simulations

177 To determine the ability of B_{vi} and B_{vv} to predict plume onset, a suite of
178 two-phase numerical simulations of particle settling in water was performed
179 using a multiphase computational fluid dynamics code called Fluidity, vary-
180 ing the particle diameter and constant particle mass flux (into the water from
181 above) over a range that encompassed the laboratory particle settling exper-
182 iments of Carey (1997). The size of the water tank in the simulations was
183 $0.3 \text{ m} \times 0.3 \text{ m} \times 0.7 \text{ m}$, replicating the geometry of Carey’s experiments.

184 Initially, no particles were present in the domain, except along the surface
185 where random perturbations in the particle volume fraction were introduced
186 such that $10^{-7} \leq \alpha_p \leq 10^{-5}$. This essentially ‘seeded’ instabilities in the
187 growing particle-water layer so that plumes could form. For numerical rea-
188 sons, α_p was bounded below by a value of 10^{-7} instead of zero to avoid
189 singularities in the system of linear equations. The velocity of both phases,
190 denoted \mathbf{u}_f and \mathbf{u}_p respectively, was set to $\mathbf{0} \text{ ms}^{-1}$ (where $\mathbf{0}$ is the zero vector)
191 at $t = 0 \text{ s}$. Throughout the simulations, no-normal flow conditions $\mathbf{u}_f \cdot \mathbf{n} = 0$
192 and $\mathbf{u}_p \cdot \mathbf{n} = 0$ (where \mathbf{n} is the normal vector) were enforced along each bound-
193 ary of the domain to prevent the fluid and particles from exiting. Particles
194 entered the domain through the top boundary at a constant user-specified
195 mass flux rate (defined later).

196 The following physical parameters were used and remained constant through-
197 out all simulations: $\rho_p = 2,340 \text{ kgm}^{-3}$, $\rho_f = 1,000 \text{ kgm}^{-3}$, $\mu_f = 0.001 \text{ Pas}$,
198 and $g = 9.8 \text{ ms}^{-2}$. The particle phase was assumed to be inviscid such that
199 $\mu_p = 0 \text{ Pas}$. The range of mass flux was $2.50 \times 10^{-4} - 6.11 \times 10^{-4} \text{ kgm}^{-2}\text{s}^{-1}$
200 (the range determined for the eruption of Mount St Helens on 18 May 1980

201 (Sarna-Wojcicki et al., 1981; Scheidegger et al., 1982; Carey, 1997)), and d_p
202 ranged between 20 and 64 μm as per the experiments by Carey (1997). In
203 total, four different mass fluxes and six different particle diameters within
204 these ranges were chosen, detailed in Table 1.

205 The domain was discretised using an unstructured mesh of solution nodes,
206 composed of triangular and tetrahedral elements in two and three dimensions
207 respectively, produced by Gmsh (Geuzaine and Remacle, 2009). The char-
208 acteristic element length was fixed at 0.0025 m, except in the preliminary
209 three-dimensional simulation mentioned in the next paragraph which used
210 mesh adaptivity (Piggott et al., 2008) to optimise the mesh throughout the
211 simulation and place high resolution only where necessary in order to reduce
212 computational costs; in this case, the upper and lower bounds on the ele-
213 ment length were set to 0.1 m and 10^{-5} m respectively (Jacobs, 2013). The
214 spatial discretisation of the model equations was performed using a Galerkin
215 finite element method for the continuity and momentum equations, and a
216 control volume method for the volume fraction fields (Jacobs et al., 2013; Ja-
217 cobs, 2013). The implicit backward Euler method was used for the temporal
218 discretisation, in conjunction with an adaptive time-stepping scheme which
219 maximised the time-step subject to a Courant number of 0.5. All simulations
220 were performed until $t = 600$ s, which was enough time for plumes to form
221 for all combinations of particle diameters and mass fluxes.

222 To establish any possible effect of problem geometry on plume formation,
223 both 2D and 3D simulations were first performed using $d_p = 48 \mu\text{m}$ and
224 a mass flux of $4.72 \times 10^{-4} \text{ kgm}^{-2}\text{s}^{-1}$ (see Figure 2). In both cases, initial
225 particle settling happened individually at the appropriate Stokes' law veloc-

Reference	Mass flux ($\text{kgm}^{-2}\text{s}^{-1}$)	d_p (μm)
A1 – A6	2.50×10^{-4}	26, 32, 40, 48, 56, 64
B1 – B6	3.61×10^{-4}	26, 32, 40, 48, 56, 64
C1 – C6	4.72×10^{-4}	26, 32, 40, 48, 56, 64
D1 – D6	6.11×10^{-4}	26, 32, 40, 48, 56, 64
E1 – E2	4.72×10^{-4}	26, 48

Table 1: Reference table for the 24 simulations in the numerical parameter study (A1 – A6, B1 – B6, C1 – C6 and D1 – D6), and for the experimental data points (E1 – E2).

ity, forming a uniform layer of thickness h . Eventually, instabilities at the base of this layer grew into plumes that settled to the base of the tank much more rapidly than the initial, individual particle settling speed. The layer thickness, particle volume fraction and time at the onset of plume formation differed by less than 10% between the 2D and 3D simulations. Therefore, for computational expedience, only 2D simulations were performed for the remaining particle diameters and mass fluxes. Note that for some simulations the nominal $0.3 \text{ m} \times 0.7 \text{ m}$ domain was extended in the vertical direction to accommodate plumes that grew longer than 0.7 m.

To quantify the conditions at the onset of plume formation and hence evaluate the accuracy of the dimensionless quantities for predicting plume onset (B values), the values of h and δ needed to be extracted from the simulation results. By assuming that particles in the layer settle under Stokes' law (at least until plumes have formed), the layer thickness h was consistently found using the Stokes' law settling velocity multiplied by the time at the onset of pluming. This assumption was tested *a posteriori* and shown to be

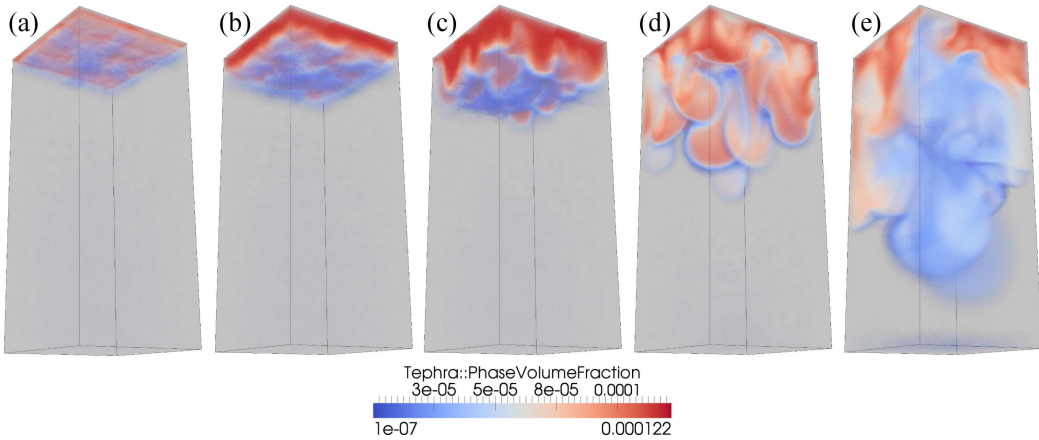


Figure 2: Three-dimensional simulation of particles settling through a tank of water at $t =$ (a) 10, (b) 30, (c) 50, (d) 80, and (e) 120 s. All visualisations show the whole $0.3 \text{ m} \times 0.3 \text{ m} \times 0.7 \text{ m}$ domain.

242 valid across all simulations. At a given time, the head of the growing plume
 243 of greatest amplitude δ was calculated by finding the lowermost position of
 244 the 10^{-5} particle volume fraction contour. This contour was chosen *a pos-*
 245 *teriori* as a sensible lower bound on the volume fraction of particles in the
 246 layer. The amplitude δ was then computed by taking the difference between
 247 the depth of the layer and the position of the plume head.

248 As one might expect, there is a certain amount of ambiguity involved
 249 when deciding when an instability is developed enough to be defined as a
 250 plume. Since the amplitude of a growing instability is known to be a function
 251 of the layer thickness (Manville and Wilson, 2004), this work defined the onset
 252 of pluming as the moment when $\delta = h$. The validity of this choice is discussed
 253 in Section 5. At this time, the quantities h and α_p were determined, and the
 254 dimensionless numbers B_{vv} and B_{vi} were calculated.

255 **4. Experimental Data**

256 The experiments performed by Carey (1997) used ultrasound imaging to
 257 track particle positions, which did not permit the accurate measurement of
 258 the parameters h and α_p . Some assumptions were therefore made in order
 259 to calculate estimates for experimental values of B_{vv} and B_{vi} for compari-
 260 son with the numerical simulations. Assuming that particles in the growing
 261 particle-laden layer settled at Stokes' law velocity, $\mathbf{u}_p = \mathbf{u}_{\text{stokes}}$, the distance
 262 the particles had travelled at the time of plume onset, t_{onset} , provided an
 263 approximation for the layer thickness:

$$h \approx |\mathbf{u}_{\text{stokes}}| t_{\text{onset}}. \quad (8)$$

264 Furthermore, assuming the volume fraction of particles in the layer was uni-
 265 form (because of the constant mass flux), and the total volume of the layer
 266 (including the water) was given by

$$V_{\text{layer}} = hA, \quad (9)$$

267 where A is the area through which particles fluxed in ($A = 0.9 \text{ m}^2$ for these
 268 particular experiments), then

$$\alpha_p = \frac{V_p}{V_{\text{layer}}}, \quad (10)$$

269 where V_p is the volume occupied by the particles. The mass flux of particles
 270 per unit area, \dot{M}_p , was used to calculate the volumetric flux per unit area \dot{V}_p
 271 using

$$\dot{V}_p = \frac{\dot{M}_p}{\rho_p}. \quad (11)$$

272 From this, the volume of the particle phase in the layer was calculated as

$$V_p = A\dot{V}_p t_{\text{onset}}, \quad (12)$$

273 and the volume fraction followed from

$$\alpha_p = \frac{V_p}{V_{\text{layer}}}. \quad (13)$$

274 Carey (1997) noted that plumes had formed after approximately 30 s in
 275 experiment 96-5 which used 20–32 μm diameter particles, and after approx-
 276 imately 60 s in experiment 96-1 which used 32–64 μm diameter particles.
 277 These times were used as approximations to t_{onset} for the purpose of estimat-
 278 ing B_{vv} and B_{vi} , giving two data points for each measure, denoted E1 and
 279 E2 (see Table 1).

280 5. Evaluation of the Measures

281 The results from the parameter study reinforced the expected relation-
 282 ship between the particle diameter, mass flux and layer instability. Smaller
 283 particle sizes decrease the time required for plume onset because the slower
 284 Stokes' law settling results in a higher average particle concentration in the
 285 near-surface layer. This behaviour was also witnessed in the experiments
 286 performed by Carey (1997) where, for two ranges of particle diameter (20–
 287 32 μm and 32–64 μm), there was a difference of approximately 30 s in the
 288 onset time. Similarly, a higher particle flux also causes a denser build-up of
 289 particles in the growing layer, further encouraging plume formation.

290 As expected, the calculated values of B_{vv} and B_{vi} , shown in Figures 3a
 291 and 3b respectively, are all greater than unity since the parameters h and
 292 α_p were measured at the point where plumes formed. Most importantly, the
 293 values from the measure B_{vi} (which assumes that collective settling is slowed
 294 by inertial drag) lay consistently on a particular contour (~ 1.2), whereas the
 295 values from the measure B_{vv} (which assumes that collective settling obeys
 296 Stokes' law and is therefore slowed by viscous drag) did not. In theory,
 297 one would expect plume onset to occur at a constant B value because the
 298 definition of when a plume has formed does not change between simulations.
 299 By correctly describing the drag on the plumes, the B_{vi} measure robustly
 300 estimated the timescale of collective particle settling, even when the system
 301 became more and more unstable and non-linear as a result of increasing
 302 particle diameter and flux rate. In contrast, the B_{vv} measure grossly under-
 303 estimated the timescale of collective settling.

304 Plume formation in every numerical simulation was robustly predicted
 305 by a B_{vi} value of ≈ 1.2 . This threshold value for B_{vi} was derived by defining
 306 $\delta = h$ as the condition for the onset of pluming. While the coefficient of h in
 307 this expression was chosen arbitrarily, other coefficients close to unity would
 308 still result in a consistent plume-onset B_{vi} value, but the exact threshold
 309 value would differ from 1.2. This is because for any δ proportional to h the
 310 ratio of timescales between individual and collective particle settling is the
 311 same to within a constant factor for a given plume scenario.

312 Although the estimated experimental data points do not follow an exact
 313 contour for either measure, the two experimental B_{vi} values are much more
 314 consistent than the two B_{vv} values. The small discrepancy in the B_{vi} values

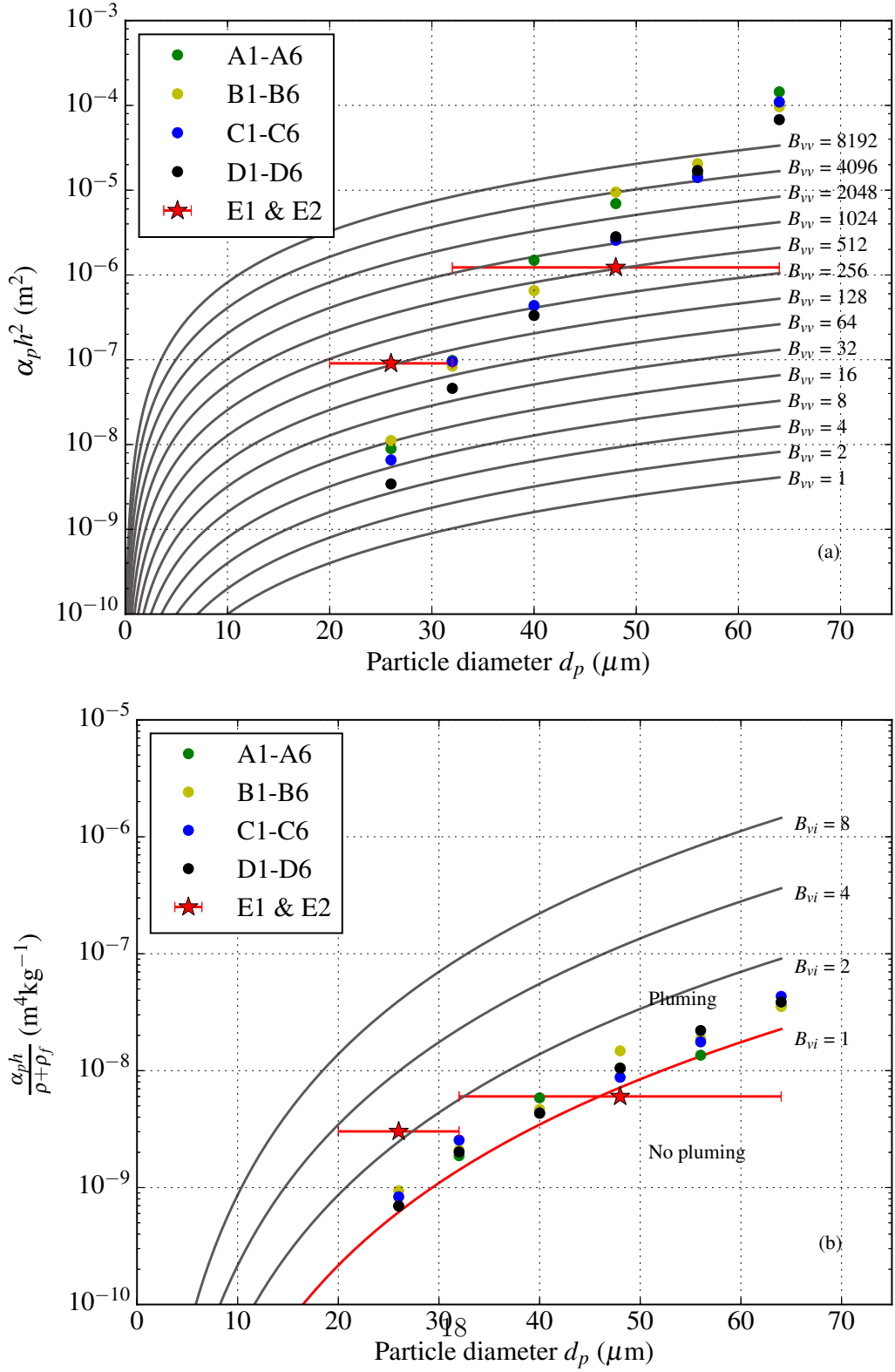


Figure 3: B_{vv} and B_{vi} results (presented in (a) and (b), respectively) using $\delta = h$. Several contours of B_{vv} and B_{vi} are given by solid lines. Due to the differences in the formulations of the measures, different quantities were considered along the y -axis. These quantities are related to the input mass flux of particles across the water surface, and are functions of the volume fraction of particles α_p , the layer thickness h , the fluid density ρ_f and the bulk density ρ .

315 is easily explained by the ambiguity in plume onset time, which could not be
316 accurately determined from the ultrasound images. Moreover, the approxi-
317 mate time of plume onset in the experiments does not necessarily correspond
318 to the point at which $\delta = h$, thus potentially introducing further uncertainty
319 in the experimental estimates.

320 At earlier times when plumes had not formed (i.e. before the point at
321 which $\delta = h$), the values of B_{vv} and B_{vi} were also calculated to show that
322 B_{vi} is less than unity, while B_{vv} is much greater than unity, demonstrating
323 the inaccuracy of the measure that assumes collective settling is slowed by
324 viscous drag. Simulation C4 is considered here for demonstration purposes
325 because the relatively low mass flux and large particle diameter favoured the
326 stability of the growing layer. Figure 2a shows the particle volume fraction
327 at $t = 10$ s. Clearly plumes had not formed at this point, and only very small
328 initial perturbations (with $\delta \ll h$) are present along the base of the layer. It
329 was found that all particles were still travelling at their Stokes' law velocity
330 at this point in time. The B_{vi} measure yielded a value less than unity (~ 0.3),
331 correctly implying that individual particle settling dominated the dynamics.
332 This also agrees with an estimated B_{vi} value of ~ 0.37 (see the contour plot
333 in Figure 4), computed using estimates for the volume fraction and layer
334 thickness as described in Section 4. However, a B_{vv} value of ~ 15 implied
335 that plumes were already well into the growth stage. This demonstrates
336 that the measure which assumes collective settling is slowed by viscous drag
337 grossly under-estimates the timescale of plume growth and descent. On the
338 other hand, the new measure which assumes collective settling is slowed by
339 inertial drag is able to more accurately measure the tendency for plumes to

340 form at early times.

341 5.1. *Alternative Formulation*

342 The measures in their current form require knowledge about the state of
343 the layer, in particular the layer thickness, the volume fraction of particles
344 within it, and (in the case of B_{vi}) the amplitude of the growing instabilities.
345 Given this information, the non-dimensional number can be used to deter-
346 mine whether plumes will form. These quantities have to be estimated in
347 practice since measuring them after or during an eruption event would be
348 infeasible or impossible. However, as an alternative to calculating B_{vv} and
349 B_{vi} directly from the state of the system, the measures can be re-formulated
350 in terms of a critical layer thickness, denoted h_{crit} . For pluming to occur,
351 the value of h must satisfy $h_{\text{crit}} < h < H$, where H is the height of the
352 water column. The thickness of the layer h can be estimated throughout
353 time using Stokes' law since the particles within the layer settle individually.
354 Furthermore, the critical value is expressed only in terms of the volumetric
355 influx of particles and the particle diameter, such that the measures can be
356 useful regardless of whether the exact values for h , δ and α_p are known.

357 By using a similar technique to that used when estimating the values
358 of B_{vv} and B_{vi} from the experiments of Carey (1997), an expression for α_p
359 (assumed to be constant and uniform in the layer) was formulated:

$$\alpha_p = \frac{\dot{V}_p}{|\mathbf{u}_{\text{stokes}}|}, \quad (14)$$

360 where \dot{V}_p is the volumetric flux (per unit area) and $\mathbf{u}_{\text{stokes}}$ is the Stokes' law
361 velocity. This was then used to re-arrange both measures in terms of h , and

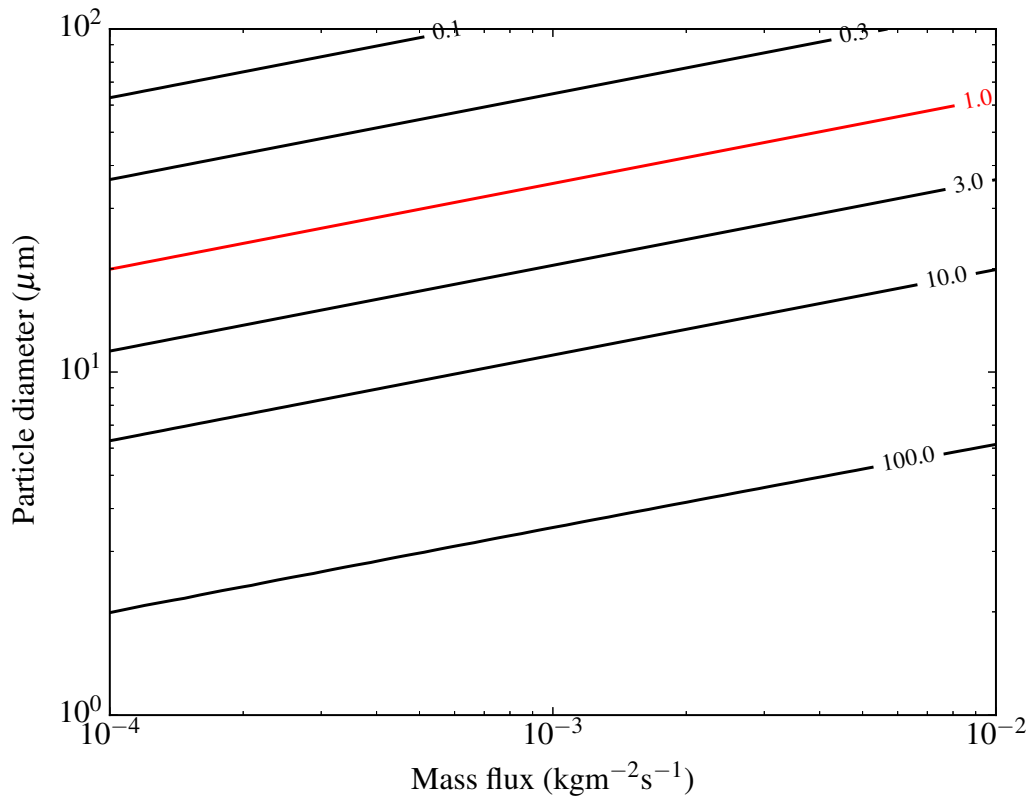


Figure 4: A contour plot of B_{vi} , computed using estimates for the particle volume fraction and layer thickness, at $t = 10$ s for various mass fluxes and particle diameters. The line $B_{vi} = 1$ is highlighted in red. The plot further reinforces the finding that higher mass flux and/or smaller particle diameter encourages plume formation.

362 by setting B equal to unity, h_{crit} was derived. For B_{vv} , the value of h_{crit} is
 363 given by

$$h_{\text{crit}} = \sqrt{\frac{d_p^4 g (\rho_p - \rho_f)}{18 \mu_f \dot{V}_p}}. \quad (15)$$

364 On the assumption that $\rho \approx \rho_f$ in (7) because $\alpha_f \approx 1$, and that plumes have
 365 formed when $\delta = h$, the value of h_{crit} for B_{vi} is given by

$$h_{\text{crit}} = \left(\frac{2\rho_f}{\beta} \right) \left(\frac{(\rho_p - \rho_f)^2 g^2 d_p^6}{5832 \mu_f^3 \dot{V}_p} \right). \quad (16)$$

366 Figures 5a and 5b illustrate the relationship between the particle diameter
 367 and the critical value h_{crit} for both measures, for all volumetric flux rates
 368 considered in this paper. While all the values of h_{crit} and h were such that
 369 $h_{\text{crit}} < h < H$ was satisfied, a measure could only be considered meaningful
 370 and useful if the expected h_{crit} values *consistently* agree with the actual values
 371 of h at the time of plume formation (i.e. if the values of h_{crit} run parallel
 372 to all the layer thicknesses determined from the numerical simulations). As
 373 demonstrated in Figure 5a, this is clearly not the case for the B_{vv} measure
 374 whose values for h_{crit} start to diverge from the theoretical prediction. In
 375 contrast, the values of h_{crit} obtained from the B_{vi} measure, which takes into
 376 account the inertial drag acting on the particles, run parallel to all the data
 377 points as shown in Figure 5b. This further demonstrates the robustness and
 378 applicability of the B_{vi} measure when the exact values of h , δ and α_p are not
 379 readily available.

380 Since the volumetric flux and particle diameter are two quantities that
 381 are often known during or after an eruption event, a plot of h_{crit} (for the B_{vi}
 382 measure) against the volumetric flux for various particle diameters is given

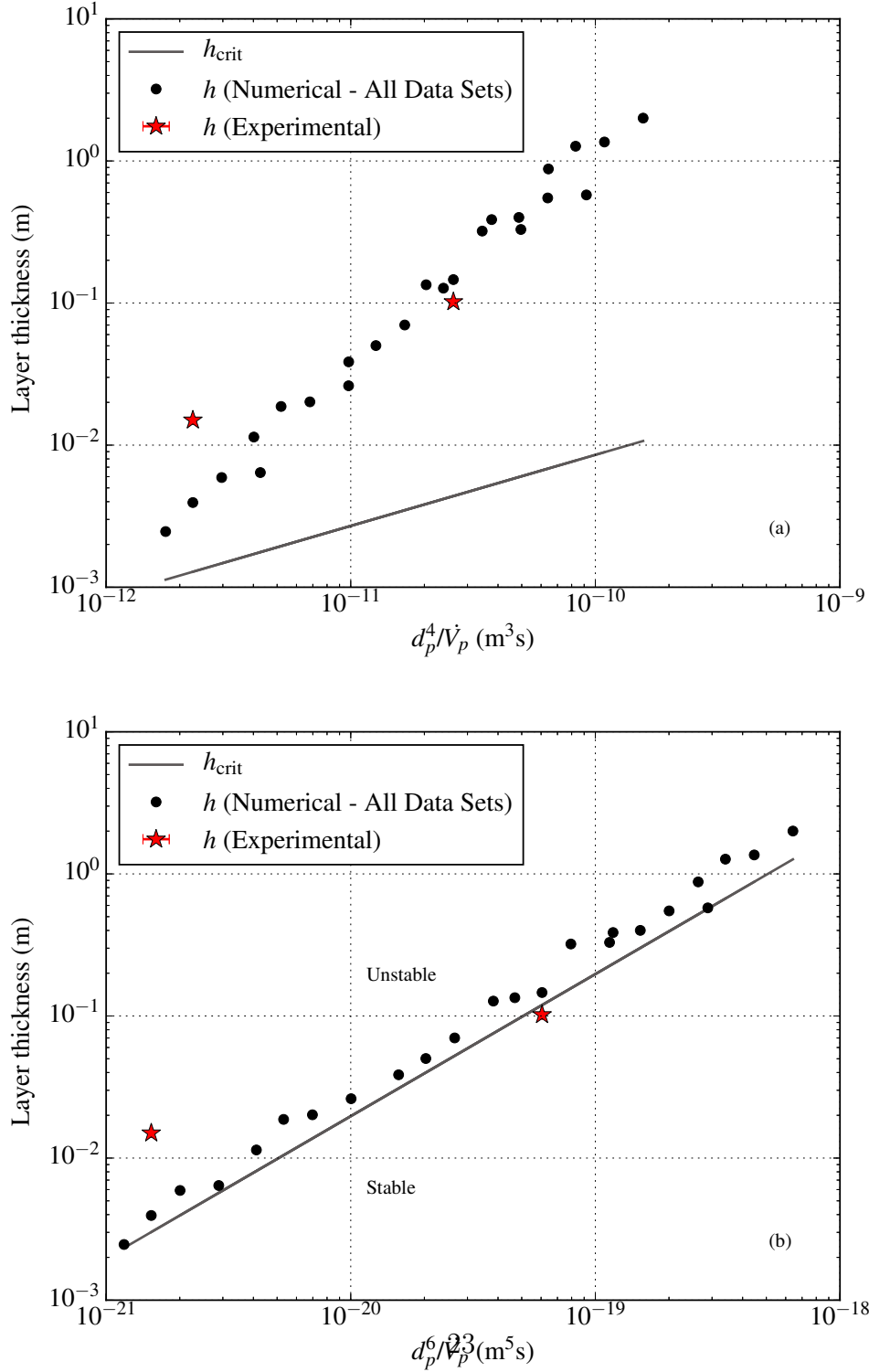


Figure 5: Plots of layer thickness h and the critical layer thickness h_{crit} for (a) the B_{vv} measure, and (b) the B_{vi} measure. The quantities d_p^4 / \dot{V}_p and d_p^6 / \dot{V}_p^3 (which are functions of the particle diameter d_p and the volumetric flux \dot{V}_p) were considered along the x -axis to allow all data points to be plotted against a single h_{crit} line (rather than having a separate line for each volumetric flux or particle diameter).

383 in Figure 6 for reference. This also helps to demonstrate once again how
384 increasing the volumetric flux rate and/or decreasing the particle diameter
385 makes the system increasingly unstable, as shown by the smaller h_{crit} values.

386 In the context of subaqueous explosive volcanic eruptions, in which the
387 near-surface layer is formed from particles being forced upwards, high inertia
388 and buoyancy are necessary to sustain particle ascent before the eruption
389 column spreads out laterally along the water’s surface (White, 2000; White
390 et al. (2003), pp. 9–12). If the mass flux of particles at the surface is greater
391 than that typically achieved by atmospheric ash fallout, then Figure 6 im-
392 plies that a much thinner layer will be required to initiate plume onset (for
393 a given particle diameter). It is also important to note that, since plume
394 size is related to h and therefore h_{crit} , any eruption column that is unable
395 to sustain its upward motion and is thicker than h_{crit} will collapse as a den-
396 sity current/plume, regardless of whether the ash particles reach the water’s
397 surface.

398 *5.2. Including Additional Particle Sizes*

399 All the simulations presented thus far have considered multiphase flows
400 comprising ash particles of the same diameter, known as monodisperse flows.
401 Such flows are certainly an idealisation since real volcanic ash particles can
402 vary greatly in diameter (Rose and Durant, 2009). The inclusion of addi-
403 tional particle phases each defined by a different particle diameter, forming
404 a so-called polydisperse flow (Crowe et al. (1998), p. 37), can therefore sig-
405 nificantly alter the behaviour and enhance the realism of the results. To
406 investigate the effect of multiple particle diameters on the transport of ash in
407 water, and to determine how the theoretical measures defined earlier should

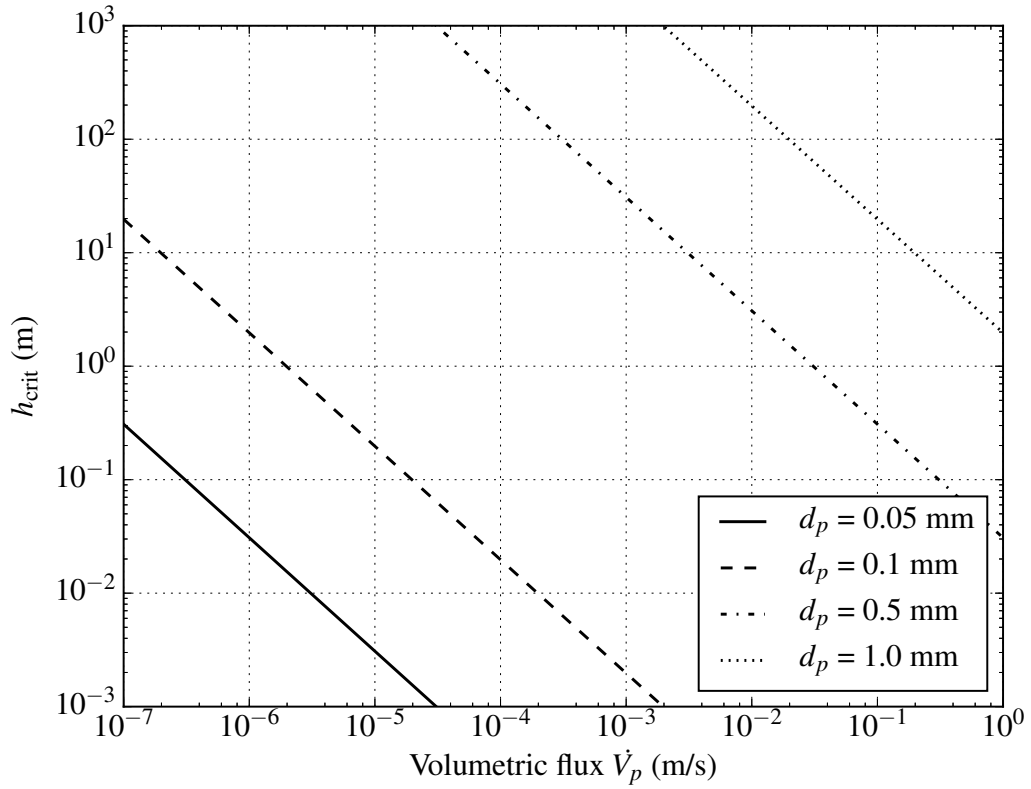


Figure 6: Plot of critical layer thickness h_{crit} (from the B_{vi} measure) against volumetric flux \dot{V}_p , for various particle diameters. The volumetric fluxes (per unit area) used in the experiments by Carey (1997) are of $O(10^{-7})$ ms^{-1} .

408 be modified to support polydisperse flows, a three-phase simulation was set
409 up in Fluidity which extended the earlier two-phase simulations.

410 Two particle diameters $d_{p_1} = 26 \mu\text{m}$ and $d_{p_2} = 48 \mu\text{m}$ in the range of
411 those considered by Carey (1997) were employed. Both particle phases had
412 the same density of $2,340 \text{ kgm}^{-3}$. A previously used (total) mass flux of
413 $4.72 \times 10^{-4} \text{ kgm}^{-2}\text{s}^{-1}$ was chosen and remained constant, but was divided
414 equally between the two particle phases such that each one fluxed in at
415 $2.36 \times 10^{-4} \text{ kgm}^{-2}\text{s}^{-1}$. All other aspects of the set-up remained the same as
416 the earlier two-phase simulations.

417 After performing the simulation, it was found that at early times the 26
418 μm particles and 48 μm particles behaved just like their monodisperse ver-
419 sions. That is, Stokes' law settling ensued once the particles first entered
420 the water tank, as shown by the good agreement with the Stokes' law veloc-
421 ities of 0.00049 ms^{-1} and 0.00168 ms^{-1} (for $d_{p_1} = 26\mu\text{m}$ and $d_{p_2} = 48\mu\text{m}$,
422 respectively) in Figure 7. The near-surface layer of particles that formed
423 was essentially divided up into two parts as a result of the different settling
424 velocities; the smaller 26 μm particles formed their own relatively thin and
425 more concentrated 'sub-layer', while the larger 48 μm particles were able to
426 overtake the 26 μm particles and form a thicker layer as shown in Figures
427 8a and 8f. After the initial growth of the layer (as a whole), plumes formed
428 from the thinner sub-layer layer of 26 μm particles while the layer of 48 μm
429 particles remained almost uniform in shape, as shown in Figures 8b and 8g.
430 This occurred at approximately the same time as the monodisperse 26 μm
431 simulation, but the plumes grew at a slightly slower rate which may have
432 been the result of the presence of larger particles that typically increase the

433 stability of the system. Despite this small difference, the dynamics of the each
434 particle phase were qualitatively similar to the monodisperse simulations of
435 26 μm and 48 μm particles up until this point.

436 The plumes of 26 μm particles that grew from the thin sub-layer eventu-
437 ally started to influence the dynamics of the other part of the layer composed
438 solely of 48 μm particles, which were still settling at near-Stokes' law veloc-
439 ity, by entraining them. The growth of any small instabilities in the 48 μm
440 particle sub-layer was essentially over-ridden by the presence of the plumes
441 of smaller particles. Therefore, while the two particle phases behaved almost
442 independently at early times, in a similar manner to the separate monodis-
443 perse versions, it was the smaller particles in the system that influenced the
444 dynamics of the whole polydisperse system at later times.

445 As the plumes continued to grow and entrain material the two particle
446 phases became strongly coupled to one another (as shown by the similar
447 velocity profiles in Figure 7 at late times). This resulted in their volume
448 fraction fields becoming almost identical in shape (see Figures 8c–e and 8h–
449 j). The plumes were of a comparable length to those composed solely of
450 26 μm particles, although they appeared to be a few millimetres thicker as
451 a result of the larger particles. Furthermore, as the plumes descended, the
452 smaller particles tended to move a small distance away from the surface of
453 the plumes and instead drift behind a thin outer layer of larger particles
454 because of drag reduction effects. This suggests that a degree of sorting by
455 settling velocity takes place during collective particle descent and deposition,
456 which is commonly seen in the real world (Carey, 1997; Manville and Wilson,
457 2004).

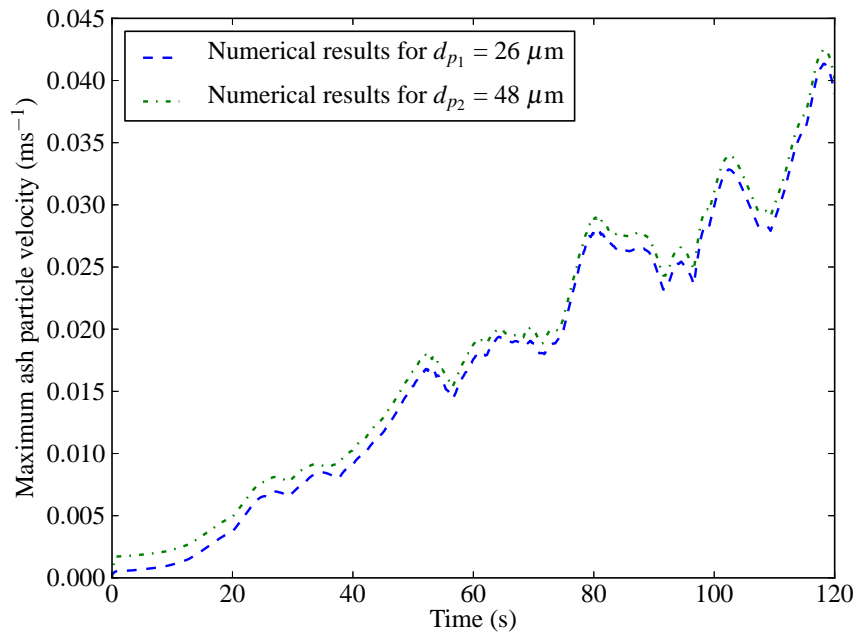


Figure 7: Maximum velocity of ash particles in each particle phase against time, with $d_{p1} = 26 \mu\text{m}$ and $d_{p2} = 48 \mu\text{m}$, in a two-dimensional polydisperse simulation of the experiments by Carey (1997).

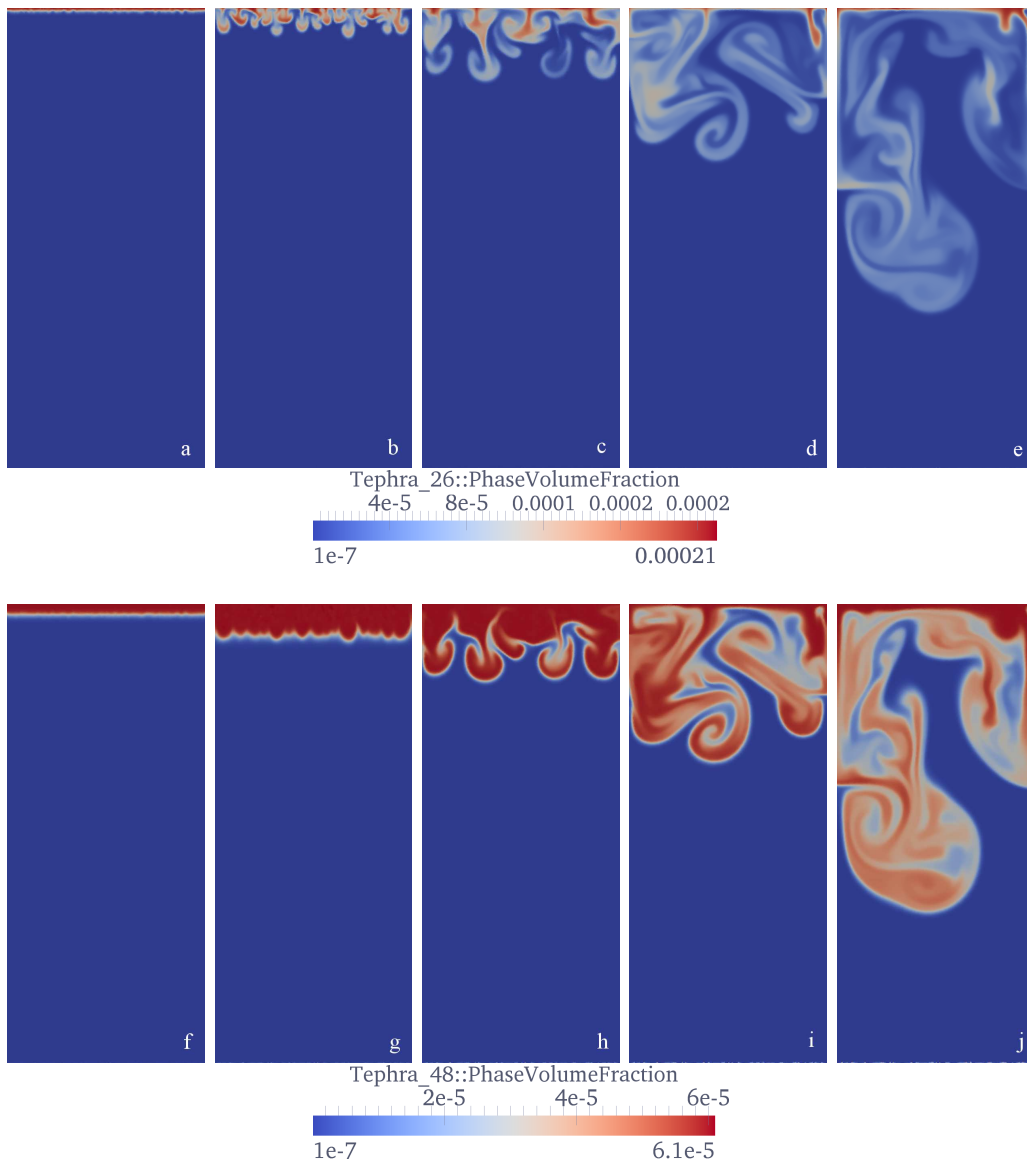


Figure 8: Visualisation of a three-phase, polydisperse ash settling simulation in Fluidity, with $d_{p_1} = 26 \mu\text{m}$ (top row) and $d_{p_2} = 48 \mu\text{m}$ (bottom row), at $t = 10, 30, 50, 80$ and 120 s (from left to right). The volume fraction of the particle phase (α_p) is shown; warmer colours represent a higher volume fraction. All visualisations show the whole $0.3 \text{ m} \times 0.7 \text{ m}$ domain.

Reference	Mass flux ($\text{kgm}^{-2}\text{s}^{-1}$)	d_{p1} (μm)	d_{p2} (μm)
P1	4.72×10^{-4}	20	26
P2	4.72×10^{-4}	26	32
P3	4.72×10^{-4}	32	48
P4	4.72×10^{-4}	48	64

Table 2: Reference table for the polydisperse simulations in the numerical parameter study.

458 Since the theoretical measures of the tendency for plume formation de-
459 pend on the particle diameter, it is worth considering how the measures
460 should be modified to support multiple particle diameters. To this end, four
461 additional polydisperse simulations were performed. The particle diameters
462 chosen covered the range used by Carey (1997) and are detailed in Table 2.

463 For the purpose of computing the dimensionless quantities B_{vv} and B_{vi} ,
464 plumes were once again said to have formed when $\delta = h$. However, the
465 calculation of the layer thickness through Stokes' law (and also the calculation
466 of $\tau_{\text{individual}}$) needs to be considered carefully. It has already been shown here
467 that the dynamics of ash settling in water can be affected heavily by the
468 end members of the particle size range, so simply using an average for d_p
469 when computing both the layer thickness and $\tau_{\text{individual}}$ may not be accurate
470 in general. It is also not appropriate to define the layer thickness as the
471 maximum of the thicknesses of the two 'sub-layers' that form within the
472 whole near-surface layer, because the thicker sub-layer (comprising larger
473 particles) will eventually become entrained within the plumes growing from
474 the shallow sub-layer (comprising smaller particles). It is because of this
475 reason that using the Stokes' law settling velocity of the smaller particles

476 instead of the larger particles gave a good estimation of the layer thickness.
477 Therefore, when computing h and $\tau_{\text{individual}}$, d_p was chosen to be equal to d_{p1} .

478 The results from the parameter study of the polydisperse simulations are
479 plotted in Figure 9 (for the B_{vi} measure only). Once again, the values for B_{vv}
480 did not lie consistently on a particular contour, whereas the measure that
481 took into account the balance between gravitational forces and inertial drag
482 (B_{vi}) did. Moreover, this particular contour was approximately the same as
483 the one from the monodisperse simulations, suggesting that the measures are
484 robust even when multiple particle sizes are considered. Note also that only
485 the definition of the layer thickness and d_p (in $\tau_{\text{individual}}$) needed to be treated
486 carefully; the formulation of the dimensionless quantity itself did not need to
487 be changed.

488 6. Discussion

489 By once again assuming that $\rho \approx \rho_f$ in (7) because $\alpha_f \approx 1$, a useful
490 property of (7) is that B_{vi} is a function of the product of h and α_p , which is
491 the volume of particles per unit area in the particle-laden water layer at the
492 onset of pluming. Assuming that material reaching the sea or lake floor by
493 plumes spreads laterally as it is deposited to form a semi-continuous layer
494 of approximately uniform thickness, mass conservation implies that the final
495 deposit should contain the same volume of particles per unit area as the
496 original particle-water layer. Hence, (7) provides a measure of the tendency
497 for plumes to form which can be calculated from the properties of the final
498 deposit: the product of the volume fraction of particles in the deposit $\alpha_{p,\text{deposit}}$
499 and the deposit thickness h_{deposit} . Knowledge of the mass flux and duration

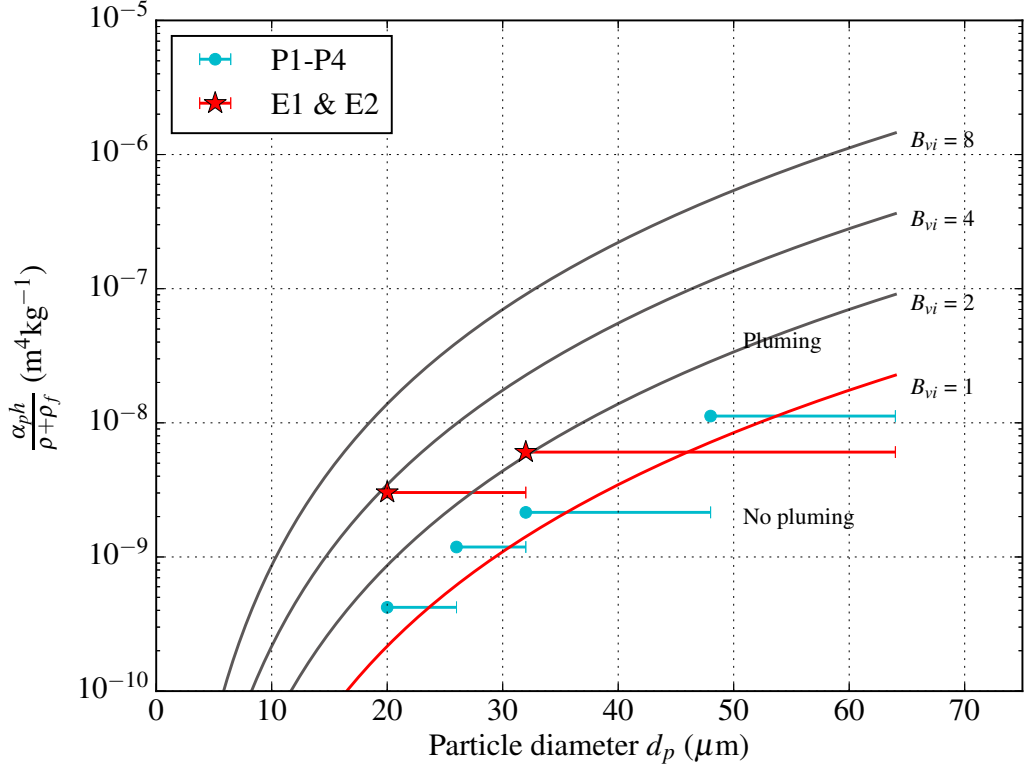


Figure 9: B_{vi} results from the four polydisperse simulations (points P1–P4, detailed in Table 2), using $\delta = h$. Several contours of B_{vi} are given by solid lines. Note that the x -coordinate of each numerical data point corresponds to the smallest particle diameter used in each polydisperse simulation, since this value is used to compute B_{vi} . As before, points E1 and E2 correspond to the experimental data; however, in light of the findings from the polydisperse simulations, the smallest particle diameters from the original experiments by Carey (1997) ($d_p = 20 \mu\text{m}$ and $d_p = 32 \mu\text{m}$) were used instead of the averages given in Table 1.

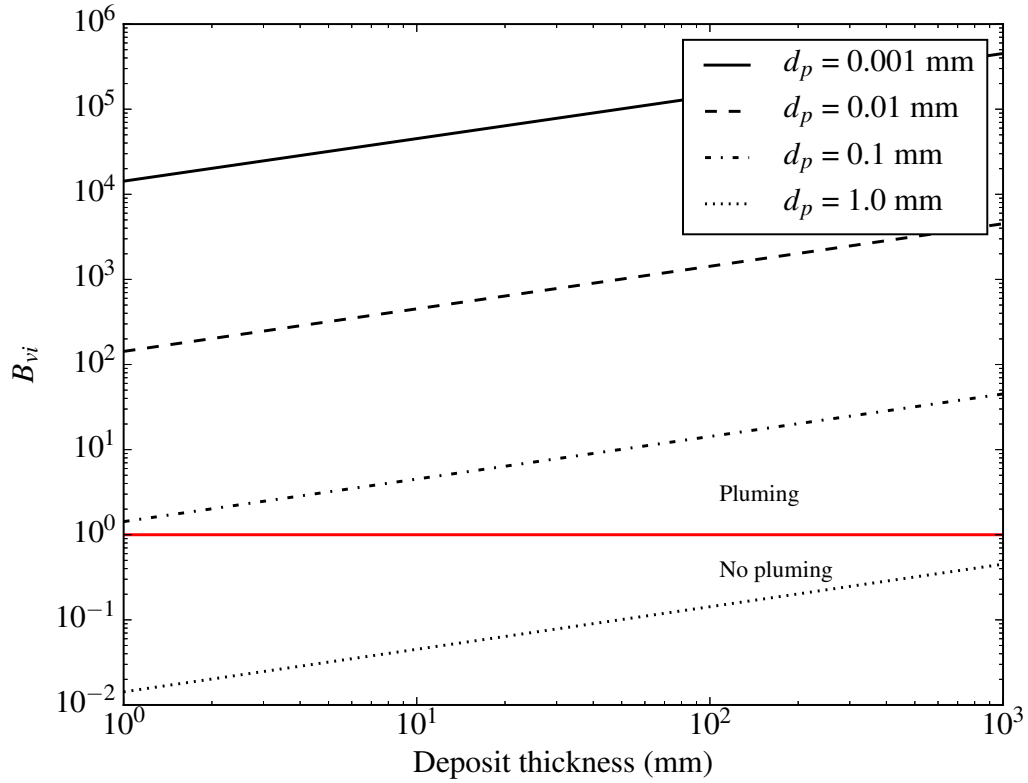


Figure 10: B_{vi} as a function of deposit thickness for various particle diameters. The horizontal red line represents $B_{vi} = 1$ and indicates the threshold for stability; values of $B_{vi} > 1$ imply that plume formation is likely.

500 are not required. Adopting this approach, Figure 10 shows how B_{vi} depends
 501 on the deposit thickness for various particle diameters. A particle volume
 502 fraction of 0.55 has been assumed for the final deposit, based on typical bulk
 503 densities of compacted wet ash (Macedonio and Costa, 2012).

504 The plot shows that for particles smaller than 0.1 mm in diameter, B_{vi}
 505 is greater than unity for final deposits thicker than 1 mm, suggesting that
 506 plume formation is expected in the formation of most benthic ash deposits,

507 particularly thick (single) deposits or those comprised of fine particles. On
 508 the other hand, a larger particle diameter helps to stabilise the system and
 509 prevent pluming. For $d_p \geq 1$ mm, B_{vi} values are less than unity for the
 510 range of deposit thicknesses considered, suggesting that in such cases the
 511 Stokes' law settling velocity is high enough to prevent a concentrated particle-
 512 water layer from building up near the surface and causing Rayleigh-Taylor
 513 instabilities to develop.

514 The Reynolds number is a useful dimensionless quantity for determining
 515 whether viscous or inertial drag effects dominate the dynamics. By defining
 516 separate Reynolds numbers for individual particles and plumes, one can de-
 517 cide which B measure is appropriate for a given geophysical scenario. These
 518 Reynolds numbers are respectively defined as

$$\text{Re}_{\text{particle}} = \frac{\rho_f |\mathbf{u}_p| d_p}{\mu_f}, \quad (17)$$

519 and

$$\text{Re}_{\text{plume}} = \frac{\rho_f |\mathbf{u}_{\text{plume}}| d_{\text{plume}}}{\mu_f}, \quad (18)$$

520 where $|\mathbf{u}_{\text{plume}}|$ and d_{plume} are the velocity and length scale of the plume.

521 The B_{vi} measure is appropriate in cases where $\text{Re}_{\text{particle}}$ is small and
 522 Re_{plume} is typically much greater than unity, implying that individual particle
 523 and plume settling are dominated by viscous and inertial drag, respectively.
 524 For micrometre-sized ash particles settling in water with a velocity that obeys
 525 Stokes' law, this is clearly the case for $\text{Re}_{\text{particle}}$ (e.g. $O(10^{-2})$ for the $48 \mu\text{m}$
 526 particles considered here). In contrast, $\text{Re}_{\text{plume}} \gg 1$ as the plume diameter
 527 and velocity is typically several times (or even several orders of magnitude)
 528 larger than those of the individual particles, as shown by the numerical simu-

529 lations presented in this paper and the original experiments by Carey (1997).
530 The measure B_{vi} is therefore appropriate in this case.

531 In addition to ash particles settling through bodies of water, the new
532 measure B_{vi} may also be applicable to other geophysical processes which
533 have the potential to form plumes. One example is the settling of volcanic
534 ash through the atmosphere following an explosive volcanic eruption event.
535 For small pyroclasts, Re_{particle} will still be less than unity (typically between
536 $O(1)$ and $O(10^{-5})$ for fine ash of the same size and a similar density to that
537 considered here (Bonadonna et al., 1998)) if Stokes' law continues to hold,
538 while the sheer diameter (tens to hundreds of metres) and settling velocity of
539 the growing plumes results in Re_{plume} becoming large enough to imply that
540 inertial drag forces dominate the plume's dynamics. However, it is important
541 to note that the individual descent of larger pyroclasts will be controlled by
542 inertial rather than viscous drag as a result of their size. Furthermore, unlike
543 the particles settling in water, individual particles may begin their descent
544 with a high inertia. This is certainly the case for impact ejecta re-entering
545 the atmosphere, for example. In these cases, Stokes' law will no longer hold
546 and the B_{ii} measure given in Appendix B may be more appropriate.

547 The process of crystals settling in a magma chamber is another example
548 of where a different measure is necessary (Marsh, 1988). Here, the dynamics
549 of the particles will obey Stokes' law regardless of whether they settle in-
550 dividually or collectively due to the high viscosity of the ambient fluid. In
551 this scenario, the B_{vv} measure would be more appropriate. However, unlike
552 water or air, any significant variation in the viscosity of the magma would
553 need to be taken into account.

554 7. Conclusion

555 This paper presented a new measure of the tendency for volcanic ash
556 particle plumes to form in water which, unlike existing measures, takes into
557 account the fact that plume growth and descent are controlled by the bal-
558 ance between gravitational forces and inertial (rather than viscous) drag.
559 The measure was evaluated, along with a measure by Marsh (1988) that
560 assumes Stokes' law-based (i.e. viscous drag-controlled) collective settling,
561 using results from a suite of particle settling simulations and previous ana-
562 logue experiments by Carey (1997). The measure that assumes collective
563 settling is slowed by viscous drag (B_{vv}) did not consistently predict the on-
564 set of pluming and in some cases grossly under-estimated the timescale of
565 collective particle settling. In contrast, the new measure that assumes collec-
566 tive settling is slowed by inertial drag (B_{vi}) correctly predicted plume onset
567 conditions for all numerical simulations, and was much more consistent with
568 experimental data, highlighting the need to take the inertial drag force into
569 account.

570 The robustness of the new measure became even more apparent when it
571 was re-arranged in terms of a critical layer thickness h_{crit} , such that the layer
572 thickness must satisfy $h_{\text{crit}} < h < H$ (where H is the height of the water
573 column) for pluming to occur. This quantity requires only the volumetric
574 flux of particles and the particle diameter to be known, and is therefore more
575 suitable in field studies. The values of h_{crit} for the B_{vv} measure did not
576 consistently agree with the layer thicknesses determined from the numerical
577 simulations, and in fact diverged away from them. This means that B_{vv} can-
578 not be used to robustly predict the tendency for plumes to form, since the

579 corresponding values of h_{crit} imply that plumes may form much sooner than
580 they actually do. In contrast, the layer thicknesses all ran parallel to the h_{crit}
581 line for the new measure as expected, further reinforcing its validity. The
582 ability of the new measure to predict plume onset accurately and consistently
583 allows the residence times and deposition rates of particles in a large body
584 of water to be determined more reliably. The measure therefore has signif-
585 icant implications for geological field studies since it permits the improved
586 interpretation of the layers of volcanoclastic material along the seabed.

587 The formulation of the new measure itself brought an additional benefit;
588 the value of B_{vi} could be estimated from the properties of the final deposit,
589 such that knowledge of the particle mass flux and duration are not required.
590 It was found that for typical fine-grained ash deposits greater than 1 mm in
591 thickness, it is likely that particles would have settled collectively as plumes.
592 However, care must be taken when using this estimation since it introduces
593 assumptions about the layer itself (e.g. uniform in thickness) which may not
594 always be justifiable in practice.

595 Despite the study focussing mainly on monodisperse systems with just
596 one particle size, it was demonstrated that the measure can also correctly
597 predict plume onset conditions for a polydisperse flow. Plume onset was
598 found to be governed by the smaller particles in such flows, so the value of
599 d_p in B_{vi} should be chosen to be the diameter of the smallest particle in the
600 system. Furthermore, it is worth noting that while the new measure was
601 only applied to situations involving volcanic ash, it is likely that it will also
602 be valid for other geophysical events involving small particles in water, such
603 as impact ejecta fallout.

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612 reviewer for their constructive feedback which greatly improved the quality
613 of this paper. AWE © Crown Owned Copyright (2012).

614 Appendix A. Notation

615 A list of notation used throughout this paper is given in Table A.3.

616 Appendix B. Derivation of the B_{ii} measure

617 At high Reynolds numbers the terminal velocity of an individual particle
618 can be approximated by balancing the inertial drag force with the buoyancy
619 force and the particle's weight:

$$\frac{1}{2}C_D A_p \rho_f |\mathbf{u}|^2 = \frac{1}{6}(\rho_p - \rho_f) g \pi d_p^3, \quad (\text{B.1})$$

620 where C_D and A_p are the drag coefficient and cross-sectional area of a spher-
621 ical particle, respectively (Batchelor (1973), pp. 233–234). Using the expres-
622 sion $A_p = \frac{1}{4}\pi d_p^2$ and re-arranging for the particle speed $|\mathbf{u}|$ gives

$$|\mathbf{u}| = \sqrt{\frac{4(\rho_p - \rho_f) g d_p}{3C_D \rho_f}}, \quad (\text{B.2})$$

Notation	Units	Description
t	s	Time
t_{onset}	s	Time of plume onset
$\tau_{\text{individual}}$	s	Timescale of individual particle settling
$\tau_{\text{collective}}$	s	Timescale of collective particle settling
α_p	Dimensionless	Volume fraction of the particles
α_f	Dimensionless	Volume fraction of the fluid
ρ_p	kgm^{-3}	Density of the particles
ρ_f	kgm^{-3}	Density of the fluid
ρ	kgm^{-3}	Bulk density ($\rho = \alpha_f \rho_f + \alpha_p \rho_p$)
\mathbf{u}_p	ms^{-1}	Velocity of the particles
\mathbf{u}_f	ms^{-1}	Velocity of the fluid
μ_f	Pa s	Viscosity of the fluid
g	ms^{-2}	Acceleration due to gravity
d_p	m	Diameter of the particles
Re	Dimensionless	Reynolds number
At	Dimensionless	Atwood number
h	m	Thickness of the near-surface layer
h_{crit}	m	Critical layer thickness
H	m	Height of the water column
δ	m	Maximum amplitude of the growing plumes
B_{vv}	Dimensionless	The measure by Marsh (1988)
B_{vi}	Dimensionless	The new measure presented in this paper
β	Dimensionless	Constant plume growth parameter
\dot{M}_p	$\text{kgm}^{-2}\text{s}^{-1}$	Mass flux (per unit area) of particles
\dot{V}_p	ms^{-1}	Volumetric flux (per unit area) of particles
A	m^2	Area through which particles enter the water
V_p	m^3	Volume of the near-surface layer occupied by particles
V_{layer}	m^3	Total volume of the near-surface layer (including the water)

Table A.3: The notation used throughout this paper.

623 which is similar to the expression used by Bonadonna et al. (1998) for Re
 624 > 500. It follows that the timescale of individual particle settling through a
 625 layer of thickness h is

$$\tau_{\text{individual}} = \frac{h}{\sqrt{\frac{4(\rho_p - \rho_f)gd_p}{3C_D\rho_f}}}. \quad (\text{B.3})$$

626 Finally, dividing (B.3) by the timescale for inertial drag-based collective
 627 settling:

$$\tau_{\text{collective}} = 2\sqrt{\frac{\rho_f\delta}{(\rho_p - \rho_f)\alpha_p g}}, \quad (\text{B.4})$$

628 and simplifying produces the non-dimensional number B_{ii} :

$$B_{ii} = \frac{h}{2}\sqrt{\frac{3C_D\alpha_p}{4\delta d_p}}. \quad (\text{B.5})$$

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